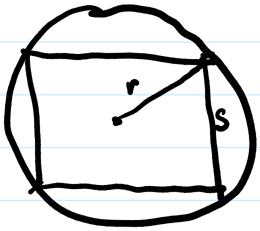
Monday, December 6, 2021 3:16 PM

Fall 2021 Math 161 Practice Problems Solutions

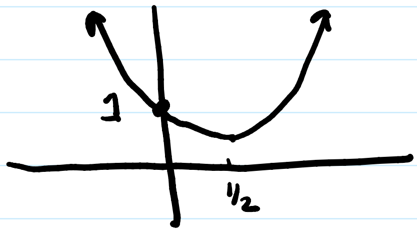
① 

$s = \sqrt{2}\, r$ by the Pythagorean Theorem

Area of square $= s^2 = 2r^2$.

② $x = t+1 \Rightarrow t = x-1$

$y = t^2+t+1 \Rightarrow y = (x-1)^2 + (x-1) + 1 = x^2 - x + 1$



③ $\lim\limits_{x\to 0} \cos(x) + \sin(x) = \cos(0) + \sin(0) = 1 + 0 = 1$

④ ⓐ $\lim\limits_{x\to 0} \dfrac{x^3}{x-\sin x} = \lim\limits_{x\to 0} \dfrac{3x^2}{1-\cos x} = \lim\limits_{x\to 0} \dfrac{6x}{\sin x} = 6$

L'Hopital L'Hopital since $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$.

ⓑ $\lim\limits_{x\to 0} \dfrac{e^x - x^2/2 - x - 1}{x^5} = \lim\limits_{x\to 0} \dfrac{e^x - 2x - 1}{5x^4} = \lim\limits_{x\to 0} \dfrac{e^x - 2}{20x^3}$

Limit Does Not exist.

⑤ $\lim\limits_{x\to 0} \dfrac{e^x(\cos \pi x + 1)}{x^4 \cos(\pi x)} = \infty$ since numerator $\to 2$ and denominator $\to 0$ and is positive.

Note: Problem 5 has a typo. Numerator should be $\cos \pi x + 1$, not $\cos \pi x - 1$

⑥ $\sin x \leq f(x) \leq \csc x$ [Recall: $\csc x = \dfrac{1}{\sin x}$]

⑥ $\sin x \le f(x) \le \csc x$ $\left[\text{Recall: } \csc x = \frac{1}{\sin x}\right]$

$$\Rightarrow \lim_{x\to\pi/2} \sin x \le \lim_{x\to\pi/2} f(x) \le \lim_{x\to\pi/2} \csc x$$

$$\Rightarrow 1 \le \lim_{x\to\pi/2} f(x) \le 1$$

, so the limit is 1 by the Squeeze Theorem.

⑦ $-1 \le \cos\left(\frac{1}{x^2}\right) \le 1$ for all x, so

$$-x^3 \le x^3\cos\left(\frac{1}{x^2}\right) \le x^3$$

$$\Rightarrow \lim_{x\to 0} x^3 \cos\left(\frac{1}{x^2}\right) = 0$$

since $\lim_{x\to 0} x^3 = \lim_{x\to 0} -x^3 = 0$

⑧
(a) DNE
(b) $7/5$
(c) $8/5$
(d) $8/5$
(e) $4/5$
(f) $7/5$
(g) $3/5$
(h) $8/5$
(i) $4/5$
(j) DNE
(k) $3/5$
(l) DNE

⑨ (a) (i) $\frac{S(3)-S(1)}{3-1} = 3$ m/s

(ii) $\frac{S(1.1)-S(1)}{1.1-1} = 2.525$ m/s

(b) $S' = v = 2 + \frac{1}{2}t$. $S'(1) = v(1) = 2.5$ m/s

⑩ $x = \frac{1}{t^2+1}$ $y = t^3+t$

$$\frac{dx}{dt} = \frac{-2t}{(t^2+1)^2} \; ; \; \frac{dy}{dt} = 3t^2+1 \; ; \; \frac{dy}{dx} = \frac{dy/dt}{dx/dt}.$$

(1)

(a) $y = (x^4 - 3x^2 + 5)^3$

$y' = 3(x^4 - 3x^2 + 5)^2 \cdot (4x^3 - 6x)$

(b) $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3}$

$y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3}$

(c) $y = \frac{3x-2}{\sqrt{2x+1}}$

$y' = \frac{3 \cdot \sqrt{2x+1} - (3x-2)\frac{1}{2}(2x+1)^{-1/2} \cdot 2}{(\sqrt{2x+1})^2}$

(d) $y = \sin^2(\cos(\sqrt{\sin(\pi x)}))$

$y' = 2\sin(\cos(\sqrt{\sin \pi x})) \cdot \cos(\cos(\sqrt{\sin \pi x})) \cdot (-\sin(\sqrt{\sin(\pi x)})) \times$

$\times \frac{1}{2}(\sin \pi x)^{-1/2} \cdot \cos \pi x \cdot \pi.$

(e) $\sin(xy) = x^2 - y$

$\cos(xy) \cdot [1 \cdot y + x y'] = 2x - y'$

$\Rightarrow \quad y' = \frac{-y\cos(xy) + 2x}{x\cos(xy) + 1}$

(f) $x \tan(y) = y - 1$

$1 \cdot \tan y + x \cdot \sec^2 y \cdot y' = y'$

$\Rightarrow \quad y' = \frac{-\tan y}{x\sec^2 y - 1}.$

$$\Rightarrow y = \frac{0}{x\sec^2 y - 1}.$$

(g) $y = \ln\sqrt{\frac{x^2-4}{x^2+4}} = \frac{1}{2}\ln(x^2-4) - \frac{1}{2}\ln(x^2+4)$

$$\Rightarrow y' = \frac{x}{x^2-4} - \frac{x}{x^2+4}.$$

(12) $y = 4\sin^2 x$

$y' = 8\sin x \cdot \cos x$

At $(\pi/6, 1)$, $y' = 8 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

Tangent line has equation: $y - 1 = 2\sqrt{3}(x - \pi/6)$

$y = 2\sqrt{3}x + (1 - \frac{\sqrt{3}}{3}\pi)$.

(13) $x^2y^2 + xy = 2$

First find $\frac{dy}{dx}$:

$2xy^2 + x^2 \cdot 2yy' + 1 \cdot y + xy' = 0$

$$\Rightarrow y' = \frac{-2xy^2 - y}{2x^2y + x} = \frac{-y(2xy+1)}{x(2xy+1)} = \frac{-y}{x}$$

$y' = 1 \Rightarrow y = -x$

$\Rightarrow x^2(-x)^2 + x(-x) = 2$

$\Rightarrow x^4 - x^2 = 2$

$\Rightarrow x^4 - x^2 - 2 = 0$

$\Rightarrow \quad x - x - 2 = 0$

$\Rightarrow \quad (x^2 - 2)(x^2 + 1) = 0$

$\Rightarrow \quad x = \pm\sqrt{2}$

$x = \sqrt{2} \Rightarrow y = -\sqrt{2}$ the points are $(\sqrt{2}, -\sqrt{2})$

$x = -\sqrt{2} \Rightarrow y = \sqrt{2}$. $(-\sqrt{2}, \sqrt{2})$

(14) Let $V(t)$ be the volume of gravel at time t.

$V = \frac{1}{3}\pi r^2 h$ where r = radius h = height.

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2$ since the diameter and height are equal.

$\Rightarrow \frac{dV}{dt} = \frac{\pi}{6} h \frac{dh}{dt}$.

$30 = \frac{\pi}{6} \cdot 10 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{18}{\pi}$ ft/min.

(15) $f(x) = xe^{2x} \xrightarrow{x=1} e^2$

$f'(x) = e^{2x} + 2xe^{2x} \xrightarrow{x=1} 3e^2$

$f''(x) = 4e^{2x} + 4xe^{2x} \xrightarrow{x=1} 8e^2$

$P_1(x) = e^2 + 3e^2(x-1)$; $P_2(x) = e^2 + 3e^2(x-1) + \frac{8e^2}{2!}(x-1)^2$

(16) $f(x) = \sin x \xrightarrow{x=0} 0$ Use center at 0.

$f'(x) = \cos x \xrightarrow{x=0} 1$

$P_1(x) = x$, so $\sin(3) \approx 3$.

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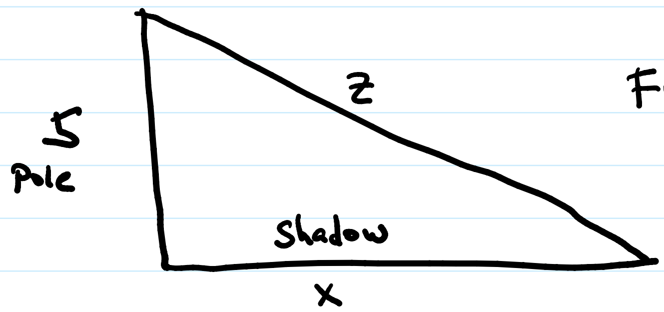

Find $\frac{dz}{dt}$ when $x=12$ given $\frac{dx}{dt} = 0.1$

Note: $z = 13$ when $x = 12$.

$$z^2 = x^2 + 25 \Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt}$$

$$\Rightarrow 2 \cdot 13 \cdot \frac{dz}{dt} = 2 \cdot 12(0.1)$$

$$\Rightarrow \frac{dz}{dt} = \frac{2.4}{26} \text{ ft/min.}$$

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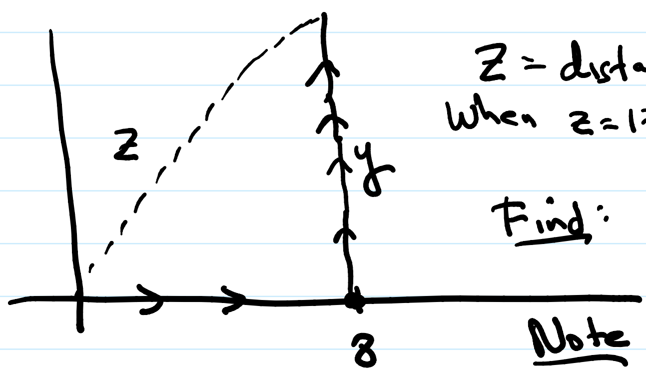


z = distance from launch

When $z = 17$, $\frac{dz}{dt} = 11$ mph

Find: $\frac{dy}{dt}$ at this instant.

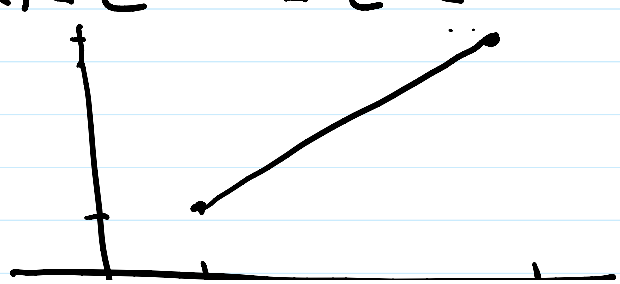
Note: $y = \sqrt{17^2 - 8^2} = 15$

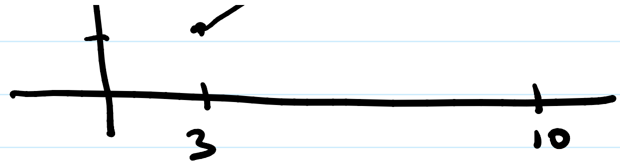
$$z^2 = y^2 + 64$$

$$2z\frac{dz}{dt} = 2y\frac{dy}{dt} \Rightarrow 17 \cdot 11 = 15\frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{17 \cdot 11}{15} \text{ mph}$$

19 $f(x) = e^{\ln x + 3} = e^3 \cdot e^{\ln x} = e^3 x$.





No critical points || Min at $x=3$ with value $3e^3$
No inflection points || Max at $x=10$ with value $10e^3$.

(20) $y = x^4 + 2x^3 - 9x^2 + 6$

$y' = 4x^3 + 6x^2 - 18x$

$y'' = 12x^2 + 12x - 18$

Local Min at $(-3, -48)$, $\left(\frac{3}{2}, \frac{-39}{16}\right)$

Local Max at $(0, 6)$

Inflection Points at $x = \frac{-1 \pm \sqrt{7}}{2}$

(21) Omit this problem.

(22)
(a) $\int_0^2 x^3 - 3x + 3 \, dx = \frac{x^4}{4} - \frac{3x^2}{2} + 3x \Big|_0^2$

$= \left(\frac{2^4}{4} - \frac{3 \cdot 2^2}{2} + 6\right) - (0)$

$= \left(\frac{\;}{4} - \frac{\;}{2} + 0\right) = $

(b) $\int_1^9 \frac{2x^2+x^2\sqrt{x}-1}{x^2}\,dx = \int_1^9 2+\sqrt{x}-\frac{1}{x^2}\,dx$

$= \left(2x+\frac{2}{3}x^{3/2}+\frac{1}{x}\right)\Big|_1^9$

$= \left(18+\frac{2}{3}9^{3/2}+\frac{1}{9}\right)-\left(2+\frac{2}{3}+1\right)$

(c) $\int \frac{-9x^2+10x}{\sqrt{3x^3-5x^2}}\,dx$ let $u=3x^3-5x^2$

$du = 9x^2-10x\,dx$

$= \int u^{-1/2}\,du = 2\left(3x^3-5x^2\right)^{1/2}+C$

(d) $\int e^x\sin(e^x)\,dx$ $u=e^x$

$du=e^x\,dx$

$= \int \sin u\,du = \cos(e^x)+C$

(e) $\int_0^3 xe^{x^2}\,dx = \frac{1}{2}e^{x^2}\Big|_0^3 = \frac{1}{2}\left(e^9-1\right)$

(f) $\int_0^{\pi/8} \sec(2\theta)\tan(2\theta)\,d\theta = \frac{1}{2}\sec(2\theta)\Big|_0^{\pi/8}$

$= \frac{1}{2}\left[\sec\frac{\pi}{4} - \sec(0)\right] = \frac{1}{2}\left[\sqrt{2}-1\right].$

23 (a) $\int_4^8 f(x)\,dx$ is bigger

(b) $\int_0^{10} f(x)\,dx \approx f(0)\Delta x + f(2)\Delta x + f(4)\Delta x + f(6)\Delta x + f(8)\Delta x$

(b) $\int_0^{10} f(x)dx \approx f(0)\Delta x + f(2)\Delta x + f(4)\Delta x + f(6)\Delta x + f(8)\Delta x$
$$= [1 + -1 + 2 + 2 + 1] \cdot 2$$

(c) $F(3) = \int_1^3 f(x)dx \approx 0$ by estimation of areas.

$F'(3) = f(3) = 2$

$F''(3) = f'(3) \approx 1.5$ by estimation of slope.

(24) $\frac{d}{dx} \int_9^{x^3} e^t(t^2 + 2t + 3) \sin t \, dt$

$= e^{x^3}(x^6 + 2x^3 + 3)(\sin x^3) \cdot 3x^2$

(25) $F(3) = 7$ $\int_3^8 F'(x)dx = 15$

$15 = \int_3^8 F'(x)dx = F(8) - F(3) \Rightarrow 15 = F(8) - 7$

$\Rightarrow F(8) = 22.$

(26) None of (a), (b), (c) is an antiderivative.

Check by finding derivatives.